EQUATION FOR THERMIONIC EMISSION IN A PLASMA

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The equation for thermionic emission from hot cathodes in the presence of a low-temperature plasma is discussed. The distribution function is obtained for the magnitude of the electric field intensity at the surface of the cathode, taking account of the effect of the individual fields of the various ions moving in the near-cathode layer. The thermionic flux density due to fluctuations of the field is found to be significantly higher than that calculated by Richardson's formula with Schottky's correction.

Richardson's classical formula with Schottky's correction for the thermionic current density

$$j_0 = AT^2 \exp\left[-\frac{e\varphi_0}{kT} + \frac{e\sqrt{eE}}{kT}\right] \tag{1}$$

where A is the thermoemission constant, T is the cathode temperature, e is the charge of the electron, φ_0 is the work function, k is Boltzmann's constant, and E is the field strength at the cathode, is applicable if the magnitude of the electric field strength is known near the surface of the cathode. In the case of electron emission in vacuo, the field E is defined as the field at a given point of the cathode acting at a given instant. In the case of electron emission in a plasma, in order to calculate the current density formula (1) is also used frequently, in which the quantity E denotes the average electric field at the surface of the cathode, found by solving Poisson's equation on McCohn's assumptions. The use of such a procedure for finding the emission current density in the presence of a plasma is very doubtful. When calculating the thermionic current by formula (1), it was proposed in [1] to take into account the individual fields of the various ions moving in the cathode region, in addition to the central field.

Between the "hot" cathode and the neutral plasma there is a region of an uncompensated space charge of ions. Ions moving toward the cathode create a fluctuating electric field on its surface, the magnitude of which at each point of the cathode depends on the number of ions near it and their disposition. We emphasize that any effect on the magnitude of the electric field at an arbitrary point of the cathode comes only from those ions whose charges are not compensated for this point, i.e., ions located in the cathode screening layer. In the case of a low-temperature plasma, when the concentration of charged particles in the range from 10^{13} to 10^{18} cm^{-3}, only a small number of ions will determine the size and direction of the electric field at an arbitrary point of the cathode; then, the fluctuations of the electric field at the surface of the cathode will be considerable and the use of a value for the central field in formula (1) is very problematical, especially as the dependence of j on E is strongly nonlinear.

In order to calculate the thermionic current density using formula (1), strictly speaking, for an arbitrary point of the cathode it is necessary to know the relation between the electric field strength and the time, E(t). If, however, E is a random quantity which at every finite instant can assume values from its minimum to its maximum E_*, then, it is possible to manage without knowing the function E(t). Such a probable approach, obviously, can be taken as acceptable as the field at an arbitrary point of the cathode is created by ions whose coordinates are random values. Thus, in order to describe the thermionic emission

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process in the plasma, it is necessary to know the distribution function of the electric field near the surface of the cathode $f(E)$.

We shall find the distribution function $f(E)$ with the following assumptions:

1) The point of the cathode to be considered is remote from the lateral boundaries of the discharge, i.e., there are no edge effects.

2) The surface of the cathode is plane and without roughness.

3) All ions recombine at the surface of the cathode.

4) The probability of finding an ion in any fixed region of the cathode space is proportional to the volume of this region, and is independent of whether or not the ions are found here.

5) The ion concentration in the cathode region is assumed to be constant.

6) The electric field strength at a given point of the cathode depends only on the location of the ion "nearest" to this point.

We locate the origin of the coordinates at the point of the cathode being considered; the axes x and y are located in the plane of the cathode, and the axis z is directed perpendicular to it on the side of the plasma.

If, at the point with the coordinates $\mathbf{r}(x, y, z)$, there is a positive ion with charge q, then, at the point (0, 0, 0), taking into account its mirror image, the ion will create a field

$$\mathbf{E} = -\frac{q\mathbf{r}}{r^2 \cdot r} - \frac{q\mathbf{r}_1}{r_1^2 r_1}$$

The absolute value of the field component along the axis z is equal to

$$E_z = \frac{2q}{r^2}\frac{z}{r}$$

Let us consider two surfaces S_1 and S_2 for which the equations have the form

$$E = 2qz\,(x^2 + y^2 + z^2)^{-3/2} \tag{2}$$

$$E - dE = 2qz\,(x^2 + y^2 + z^2)^{-3/2} \tag{3}$$

Equations (2) and (3) define a surface with constant components of the field along the axis z. The volume of a body enclosed inside the surface S_1 is equal to

$$V_1 = \frac{4\pi}{15}\left(\frac{2q}{E}\right)^{3/2} \tag{4}$$

The volume of the body enclosed between the surfaces S_1 and S_2 is equal to

$$V_2 = \frac{2\pi}{5}\,\frac{(2q)^{3/2}}{E^{5/2}}\,dE \tag{5}$$

In order that the component of the electric field strength along the axis z be in the range from E to E−dE, it is necessary to satisfy two conditions: there is no ion within the surface S_1, and there is one ion between the surfaces S_1 and S_2, i.e.,

$$p\,(E - dE \leqslant E_z \leqslant E) = f\,(E)\,dE = p_1 p_2 \tag{6}$$

where p_1 is the probability that there is no ion within the surface S_1, and p_2 is the probability that there is one ion between the surfaces S_1 and S_2.

We consider a cube, for which the center of one side coincides with the origin of the coordinates. Suppose that the characteristic dimensions of this cube are such that its volume V_0 is considerably greater than the volume V_1, i.e.,

$$V_1 / V_0 \ll 1 \tag{7}$$

The ratio V_1/V_0 becomes equal to 0.1 when $E \sim 10^4$ V/cm, i.e., with those fields when it no longer exerts any effect on thermoemission, so long as the dimensions of the cube do not exceed a value of the

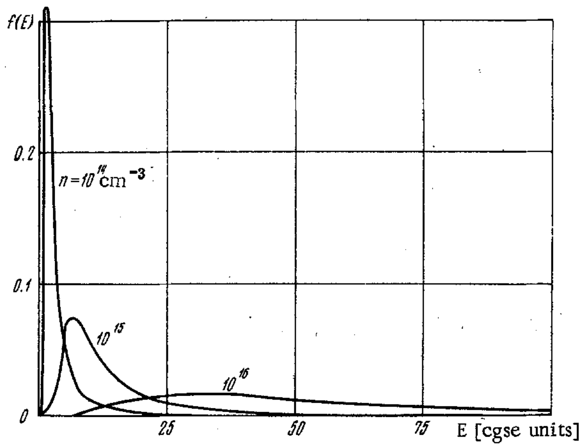

Fig. 1

order of the Debye radius; with increase of the field E, the ratio (7) will be satisfied all the more. Suppose that in the volume V_0, there are N ions. In this case, we shall consider how the probabilities p_1 and p_2 will be written

$$p_1 = (1 - V_1 / V_0)^N$$

In view of the inequality (7) and assuming that the quantity N is related with the ion concentration in the cathode region by the relation

$$n = N / V_0 \tag{8}$$

and V_1 is expressed by formula (4), we obtain

$$p_1 = \exp\left(-N\frac{V_1}{V_0}\right) = \exp\left[-\frac{4\pi n}{15}\left(\frac{2q}{E}\right)^{3/2}\right] \tag{9}$$

It is easy to show that p_2 is written in the form

$$p_2 = N\frac{V_2}{V_0}\left(1 - \frac{V_2}{V_0}\right)^{N-1}$$

Taking Eq. (5) and (8) into account, we obtain

$$p_2 = \frac{2\pi n}{5}\frac{(2q)^{3/2}}{E^{5/2}}dE\left[1 - \frac{1}{V_0}\frac{2\pi}{5}\frac{(2q)^{3/2}}{E^{5/2}}dE\right] \tag{10}$$

We rewrite relation (6), substituting the expressions for p_1 and p_2 from Eq. (9) and (10)

$$f(E)\,dE = \exp\left[-\frac{4\pi n}{15}\left(\frac{2q}{E}\right)^{3/2}\right]\frac{2\pi n}{5}\frac{(2q)^{3/2}}{E^{5/2}}dE\left[1 - \frac{2\pi}{5V_0}\frac{(2q)^{3/2}}{E^{5/2}}dE\right]$$

Neglecting terms of the second order of smallness, we obtain finally, the required distribution function

$$f(E) = \frac{2\pi n}{5}\frac{(2q)^{3/2}}{E^{5/2}}\exp\left[-\frac{4\pi n}{15}\left(\frac{2q}{E}\right)^{3/2}\right] \tag{11}$$

The function f(E) for various concentrations is shown in Fig. 1.

We note that distributions similar to Eq. (11) can be obtained from certain other relations. Thus, in cosmic astrophysics, the problem is frequently encountered where it is required to find the distribution function of the nearest neighbor to a given material point $f(r)$ with a known concentration of particles. Chandrasekar [2] obtained this distribution

$$f(r) = 4\pi n r^2 \exp(-4\pi n r^3 / 3)$$

If we consider the case of charged particles, then, at a given point the nearest neighbor will create an electric field

$$E = q / r^2$$

Then, using the device of the theory of probability, we obtain

$$f(E) = 2\pi n\frac{q^{3/2}}{E^{5/2}}\exp\left[-\frac{4\pi n}{3}\left(\frac{q}{E}\right)^{3/2}\right] \tag{12}$$

Comparison of expressions (11) and (12) shows that in the distribution (11) the term $n2\sqrt{2}/5$ occurs instead of n, i.e., in this case, there should be a reduction of the effective concentration of particles by a factor of $5/2\sqrt{2}$ (if the effect of the ion image had not been taken into account, then, a reduction of concentration by a factor of 5 would have occurred). This is explained by the fact that only the distribution of the component of the electric field strength normal to the surface of the cathode has been considered, and not the absolute value; moreover, the effect of particles disposed only for $z > 0$ was taken into account, and not over the entire space. All this leads to the effective concentration being $n_* = n2\sqrt{2}/5$ (without taking account of the image $n_* = n/5$).

We shall discuss briefly the assumptions for which the distribution function $f(E)$ is obtained. Assumption 1) is valid for thermionic cathodes with discharge currents of order 1A and higher, as the characteristic size of the spot, in this case, is somewhat larger than the thickness of the cathode layer (more precisely, the region of the uncompensated space charge of the ions). Assumption 2), generally speaking, is never strictly achieved, but it is shown in [3] that with prolonged heating of tungsten electrodes, the central field strength, due to irregularities, increases by a factor of 10 approximately in comparison with a smooth surface; on the whole, this pattern above the surface of the cathode is unchanged in the case being considered as, in contrast from the central field, the fluctuations remain the same. Assumption 3), in principle, cannot affect the nature of the distribution function; when ion reflection from the surface of the cathode is taken into account, the effective ion concentration increases in the cathode layer. Assumption 4) is achieved quite well up to a volume size comparable with the effective volume of the ion calculated in terms of the corresponding Coulomb cross section, i.e., with $V \gtrsim 10^{-21}$ cm^3. Assumption 5) should be achieved quite well at all distances when the magnitude of the cathode potential drop is less than the temperature of the ions in the body of the plasma. As, under realistic conditions, this ratio is of the order of a few units, then, the concentrations can vary by not more than a factor of 2 or 3. This is reflected somewhat in the nature of the distribution function for the region of small fields, but in the region of strong fields, there will be almost no change. On the other hand, conditions are possible when this effect may be found to be considerable. Consequently, the range of applicability of assumption 5) requires refinement. Assumption 6) is satisfied well for fields $E > 2 \cdot 10^4$ V/cm with $n > 10^{18}$ cm^{-3} as, if the distribution function of a "second" particle $f_2(E)$ be calculated, it is found that $f_2(E)/f(E) < 0.1$ when $E > 2 \cdot 10^4$ V/cm. Thus, the distribution (11) describes quite accurately the behavior of the normal component of the electric field when $n = 10^{13}$ to 10^{18} cm^{-3}. We shall now determine the average value of the thermionic current, calculated according to the distribution in Eq. (11)*

$$j = \int_0^{E^*} j_0(E) f(E)\, dE \tag{13}$$

After evaluation, we have

$$j = AT^2 \exp\left(-\frac{e\varphi_0}{kT}\right)\left[1 + \frac{4\pi nkT}{5E_*^2} \exp\left(\frac{e\sqrt{eE_*}}{kT}\right)\right]\left(1 + \frac{4kT}{e\sqrt{eE_*}}\right) \tag{14}$$

The dependence of j on the average value of the electric field strength $\langle E \rangle$ is interesting:

$$j = AT^2 \exp\left(-\frac{e\varphi_0}{kT}\right)\left[1 + \frac{3kT\langle E\rangle^{3/2}}{2\Gamma(1/3)\, e\sqrt{eE_*^2}} \exp\left(\frac{e\sqrt{eE_*}}{kT}\right)\right]\left(1 + \frac{4kT}{e\sqrt{eE_*}}\right) \tag{15}$$

where $\langle E \rangle$ is determined by the formula

$$\langle E \rangle = \int_0^{E_*} E f(E)\, dE = \left(\frac{4\pi n q^{3/2}}{15}\right)^{2/3} \Gamma(1/3) \tag{16}$$

The quantity E_* occurs in formulas (14) and (15), i.e., the maximum possible value of the field strength at a given point. It depends on the fact that at some distance r_* from the surface of the cathode an ion is still detachable from it. This was assumed to be $2 \cdot 10^{-8}$ cm in [4]. According to the estimates made in [5], this distance can be assumed to be 10^{-8} cm. In [6], a distance equal to the De Broglie wavelength is considered: $0.9 \cdot 10^{-9}$ cm. Calculations by formula (14) showed that even if we take the maximum of the values considered for r_*, which corresponds to the minimum value of E_*, then, the emission currents drawn from the cathode are found to be considerably higher than those calculated by formula (1).

Thus, in view of the impossibility of obtaining the relation $E = E(t)$ for an arbitrary point of the cathode, since the field strength there is a random quantity, it is necessary to construct the distribution function of the probability density $f(E)$, and to calculate the emission current density from the cathode by

*In the general case, the emission of electrons in a plasma of average current density can be calculated by taking into account that the energy distribution of the electrons $n(\mathscr{E})$ in the metal and the transmittance of the barrier $\mathscr{D}(\mathscr{E}, E)$ is

$$j = e\int_0^{\infty} n(\mathscr{E})\, \mathscr{D}(\mathscr{E}, E)\, d\mathscr{E}$$

formula (14). This permits an explanation of the quite high emission current obtained in the experiment in comparison with that calculated by Richardson's formula with the Schottly correction.

It can be seen from formula (14) that the emission current density is proportional to the ion concentration. This result has been confirmed qualitatively in preliminary experiments.

The plasma concentration in the cathode region has been assumed constant, but it will be necessary to find the relation between this value of the concentration and the value of the concentration in the main body of the plasma; for this, the solution of a complete closed system of equations is required for the cathode layer.

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